Design of small antennas is challenging as the Q-factor, efficiency, and radiation resistance must be controlled simultaneously. In this presentation, convex optimization together with integral expressions of the stored electromagnetic energies are used to analyze many fundamental antenna problems. The solutions to the convex optimization problems determine optimal currents, offer insight for antenna design, and present performance bounds for antennas. We present several optimization formulations such as maximal gain Q-factor quotient, minimal Q for superdirectivity, minimal Q for given far field, and efficiency. The effects of antennas embedded in structures such as mobile phones are discussed. Results are shown for various antenna geometries and compared to state of the art designs showing that many antennas perform almost optimally. A tutorial description of a method of moment implementation together with a Matlab package for convex optimization to determine optimal current distributions on arbitrarily shaped antennas is also presented.